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PROPERTIES OF MESONS DESCRIBED BY A PSEUDOSCALAR WAVE-FUNCTION

BY

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Introduction.[1]

The development of the meson theory of nuclear forces has shown the necessity to introduce in the description of the meson field two kinds of wave-functions with the invariance properties of a vector and a pseudoscalar, respectively [1] [2]. In a recent paper [3], the writer has calculated the probability of some elementary processes involving mesons which are described by a vector wave-function and which, in the following, for brevity will be referred to as *V*-mesons. Since, however, the mesons represented by a pseudoscalar function (*PS*-mesons) appear on the same footing in the description of the nuclear forces and since they seem to play a preponderant part in the cosmic radiation available for experiment [4], it is of interest to investigate different processes involving such mesons. In particular, the determination of the lifetime of *PS*-mesons, performed in Section 5, is of importance in the discussion of the connection between the decay con-

(1) The investigation reported in this paper was carried out by Dr. Tsung-Sui Chang during his stay at the Institute of Theoretical Physics at Copenhagen in 1938—39. Shortly after his departure, an account of the work was sent by Dr. Chang to the Institute, but since then connection has been interrupted due to the war. In view of the special interest which the results of Dr. Chang's work have obtained as a consequence of recent development of the meson theory, a further delay in the publication of his paper seemed, however, most undesirable and, therefore, Dr. C. Møller and Dr. S. Rozental have kindly undertaken to prepare Dr. Chang's account for print and to furnish it with an introduction containing references to the present state of the theory. Niels Bohr.

stants of radioactive elements and of free mesons in cosmic radiation [5].

The difference between the properties of mesons described by a pseudoscalar function and by a simple scalar function appears only as regards the interaction with other particles, while both these kinds of mesons behave exactly in the same way towards an electromagnetic field. The probability for a process in which *PS*-mesons are scattered by a static electric field and which is treated in Section 1 is, therefore, the same as for the scattering of mesons described by a scalar wave-function. For the processes, however, treated in Sections 2—5, which involve interaction of mesons with nucleons or with electrons and neutrinos, there is a marked difference between the scalar and the pseudoscalar field. In Section 2, we consider a process in which a meson hitting a nucleon is absorbed with emission of a light quantum; in Section 3, the elastic scattering of mesons by nucleons is calculated. Section 4 contains an investigation of a multiple process in which a meson hits a nucleon creating a pair of mesons. Finally, Section 5 is devoted to the evaluation of the lifetime of the *PS*-meson.

With the exception of the processes treated in Sections 1 and 5, the formulae deduced are valid in the case of relatively slow particles, only.

1. Elastic scattering by a static electric field.

We shall start by writing down, for reference, some of the expressions and relations referring to the pseudoscalar meson field and needed in the following calculations.

The meson field equations including the interaction with the electromagnetic field and the nucleons are

$$\varkappa^2 \varphi = \left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c} A_0\right)\chi + \left(\vec{\nabla} - \frac{ie}{\hbar c}\vec{A}\right)\vec{\xi} + 2R \qquad (1)$$

$$\vec{\xi} = \left(\vec{\nabla} - \frac{ie}{\hbar c}\vec{A}\right)\varphi - 2\vec{P} \qquad (2)$$

$$\chi = -\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c} A_0\right)\varphi + 2Q. \qquad (3)$$

Here, (x_1, x_2, x_3, ict) is the four-dimensional space vector, $(\xi_1, \xi_2, \xi_3, i\chi)$ is a complex pseudovector which, in the case of electrically charged mesons, together with the complex pseudoscalar φ is sufficient for a complete description of the meson field, (A_1, A_2, A_3, iA_0) is the electromagnetic potential vector, while $(P_1, P_2, P_3, -iQ)$ is a pseudovector, and R a pseudoscalar responsible for the interaction between the nucleons and the meson field and depending on the variables of the nucleons, only. The constant $\varkappa$ is the reciprocal of the range of the nuclear forces and connected with the mass M_m of the meson by the equation

$$\varkappa = \frac{M_m c}{\hbar}.$$

The quantities $\vec{P}$, Q, and R are given by

$$\left.\begin{aligned} \vec{P} &= \frac{f_2}{\varkappa}\psi^* \vec{\sigma}\,\frac{\tau_1 - i\tau_2}{2}\psi \\ Q &= \frac{f_2}{\varkappa}\psi^* \varrho_1 \frac{\tau_1 - i\tau_2}{2}\psi \\ R &= f_1 \psi^* \varrho_2 \frac{\tau_1 - i\tau_2}{2}\psi \end{aligned}\right\} \qquad (4)$$

where ψ is the wave-function of the nucleons, ϱ_1, ϱ_2 and $\vec{\sigma}$ are the ordinary Dirac matrices, and $\frac{1}{2}(\tau_1 - i\tau_2)$ is the operator transforming the nucleons from a neutron state to a proton state. Finally, f_1 and f_2 are the two constants of the pseudoscalar theory determining the strength of the coupling between nucleons and *PS*-mesons.

The Lagrangeian is given by

$$\frac{1}{2}\vec{\xi}\,\vec{\xi}^{\star} - \frac{1}{2}\chi\,\chi^{\star} + \frac{1}{2}\varkappa^2\varphi\,\varphi^{\star} + (\vec{P}\,\vec{\xi}^{\star} + \vec{\xi}\,\vec{P}^{\star}) + (Q\,\chi^{\star} + \chi\,Q^{\star}). \quad (5)$$

By varying $\chi^{\star}$ and $\vec{\xi}^{\star}$, while $\varphi^{\star}$ is considered as a function of $\chi^{\star}$ and $\vec{\xi}^{\star}$ given by the equation complex conjugate to (1), we get in the usual way the equations (2) and (3).

The part of the Hamiltonian which involves the meson field variables is

$$\left.\begin{array}{l} \frac{1}{2}\varkappa^2\varphi\,\varphi^{\star} + \frac{1}{2}\chi\chi^{\star} + \frac{1}{2}(\nabla\varphi)\,(\nabla\varphi^{\star}) \\ + \frac{1}{2}\frac{ie}{\hbar c}(A_0(\varphi\,\chi^{\star} - \varphi^{\star}\chi) + \frac{1}{2}\frac{ie}{\hbar c}\left[\varphi^{\star}(\vec{A}\cdot\nabla\varphi) - \varphi(\vec{A}\cdot\nabla\varphi^{\star})\right] \\ -(\varphi^{\star}R + R^{\star}\varphi) - (Q\,\chi^{\star} + Q^{\star}\chi) - (\vec{P}\cdot\nabla\varphi^{\star} + \vec{P}^{\star}\cdot\nabla\varphi) + \\ \qquad + \frac{ie}{\hbar c}\left[(\vec{A}\cdot\vec{P}^{\star})\,\varphi - (\vec{A}\,\vec{P})\,\varphi^{\star}\right]. \end{array}\right\} \quad (6)$$

The commutation rules for the field variables are

$$\left[\chi(x),\ \varphi^{\star}(x')\right] = \left[\chi^{\star}(x),\ \varphi(x')\right] = 2\,i\,\hbar\,c\,\delta(x - x') \quad (7)$$

and for the Fourier expansion of these quantities we have

$$\left.\begin{array}{l} \chi = \sum_{\vec{k}} i\sqrt{\hbar c}\,(k^2 + \varkappa^2)^{\frac{1}{4}}\left[u(\vec{k})\,e^{i(\vec{k}\vec{x})} - v^{\star}(\vec{k})\,e^{-i(\vec{k}\vec{x})}\right] \\ \chi^{\star} = \sum_{\vec{k}} -i\sqrt{\hbar c}\,(k^2 + \varkappa^2)^{\frac{1}{4}}\left[u^{\star}(\vec{k})\,e^{-i(\vec{k}\vec{x})} - v(\vec{k})\,e^{i(\vec{k}\vec{x})}\right] \\ \varphi = \sum_{\vec{k}} \sqrt{\hbar c}\,(k^2 + \varkappa^2)^{-\frac{1}{4}}\left[u(\vec{k})\,e^{i(\vec{k}\vec{x})} + v^{\star}(\vec{k})\,e^{-i(\vec{k}\vec{x})}\right] \\ \varphi^{\star} = \sum_{\vec{k}} \sqrt{\hbar c}\,(k^2 + \varkappa^2)^{-\frac{1}{4}}\left[u^{\star}(\vec{k})\,e^{-i(\vec{k}\vec{x})} + v(\vec{k})\,e^{i(\vec{k}\vec{x})}\right] \end{array}\right\} \quad (8)$$

where $u(\vec{k})$, $u^{\star}(\vec{k})$, $v(\vec{k})$ and $v^{\star}(\vec{k})$ are the operators corresponding to annihilation and creation of positive and negative mesons with momentum $\vec{k}\hbar$. The summation is extended over all values of $\vec{k}$ for which the function $e^{i(\vec{k}\vec{x})}$ has a whole number of periods between two opposite faces of a big cube with the volume L^3. In (8) and in the following formulae we have, for simplicity, put $L^3 = 1$.

For elastic scattering of *PS*-mesons by a static electric field, the only important terms of the Hamiltonian are

$$\frac{1}{2}\varkappa^2 \varphi \varphi^\star + \frac{1}{2}\chi \chi^\star + \frac{1}{2}(\vec{\nabla}\varphi)(\vec{\nabla}\varphi^\star) + \frac{1}{2}\frac{ie}{\hbar c}A_0(\varphi\chi^\star - \varphi^\star\chi) \qquad (9)$$

where the last term may be considered as a perturbation leading to a scattering process, in which the incident meson is annihilated and a new meson is created. Let us suppose that all the incident mesons are of negative charge and of momentum $\vec{k}_0\hbar$. Further, let $\vec{k}\hbar$ be the momentum of the scattered meson, and

$$\left.\begin{aligned} \varepsilon_0 &= \hbar c\sqrt{k_0^2 + \varkappa^2} \\ \varepsilon &= \hbar c\sqrt{k_0^2 + \varkappa^2} \end{aligned}\right\} \qquad (10)$$

be the energies of the incident and the scattered meson, respectively. Making use of (8), we easily find that the matrix element corresponding to the process in question is given by the expression

$$-\sqrt{n_0}\int e A_0 e^{i(\vec{k}_0 - \vec{k})\vec{x}}\, dV \cdot \frac{1}{2}\left(\sqrt{\frac{\varepsilon}{\varepsilon_0}} + \sqrt{\frac{\varepsilon_0}{\varepsilon}}\right) \qquad (11)$$

where n_0 is the number of incident mesons. Since we have conservation of energy, i. e. $\varepsilon = \varepsilon_0$, we get for this matrix element

$$-\sqrt{n_0}\int e A_0 e^{i(\vec{k}_0 - \vec{k})\vec{x}}\, dV. \qquad (12)$$

In case that the incident mesons are positively charged we get the same expression (12) but with opposite sign.

The result is exactly the same as for the scattering of mesons described by a scalar field.

2. The absorption of mesons by nucleons with emission of photons.

Let us consider a process in which a proton absorbs a negatively charged meson and simultaneously is transformed into a neutron under emission of a photon, i. e. a process given by the scheme

$$M^- + P \rightarrow N + h\nu . \tag{13 a}$$

The calculations which refer to a process involving a positively charged meson

$$M^+ + N \rightarrow P + h\nu \tag{13 b}$$

are quite analogous and lead to the same results.

Since the theory breaks down for all processes in which the de Broglie wave-length of the particles involved is very small, we shall assume that the momentum of the nucleon both before and after the process is small compared with Mc, M being the mass of the nucleon. We may, therefore, neglect the terms containing Q, R, Q^*, and R^* in the Hamiltonian. This restricts, of course, the validity of the calculations to such cases, only, in which the difference between the momenta of the meson and the photon is not too large. The remaining perturbation terms giving rise to the transition in question are now simply

$$\left.\begin{aligned} \frac{1}{2}\frac{ie}{\hbar c}\left[\varphi^*(\vec{A}\cdot\vec{\nabla}\varphi)-\varphi(\vec{A}\cdot\vec{\nabla}\varphi^*)\right]-(\vec{P}\cdot\vec{\nabla}\varphi^*+\vec{P}^*\cdot\vec{\nabla}\varphi)+ \\ +\frac{ie}{\hbar c}\left[(\vec{A}\,\vec{P}^*)\,\varphi-(\vec{A}\,\vec{P})\,\varphi^*\right]. \end{aligned}\right\} \tag{14}$$

Just as in the case of V-mesons [3], the last term leads to a direct process (13 a), while the other terms may give rise to two different second-order processes:

1) annihilation of the incident meson and creation of another negative meson M'^{-} with simultaneous emission of the photon, followed by the absorption of M'^{-} by the proton;

2) emission of a positive meson M''^{+} by the proton, followed by annihilation of M''^{+} and of the incident meson with creation of the photon.

We denote the momentum of the incident meson by $\vec{k}\hbar$, the momentum of the photon by $\vec{k}_\nu \hbar$, and the momenta and energies of the mesons M'^{-} and M''^{+} by $\vec{k}'\hbar$, $\vec{k}''\hbar$, ε' and ε''. We have, of course,

$$\left.\begin{aligned} \vec{k}' &= \vec{k} - \vec{k}_\nu \\ \vec{k}'' &= \vec{k}_\nu - \vec{k} \\ \varepsilon' &= \hbar c \sqrt{k'^2 + \varkappa^2} \\ \varepsilon'' &= \hbar c \sqrt{k''^2 + \varkappa^2}. \end{aligned}\right\} \qquad (15)$$

Through a calculation similar to that indicated in [3], we get for the matrix element corresponding to the process (13 a) the expression

$$\left.\begin{aligned} V = &-\frac{1}{\varepsilon'}\left[-i\frac{f_2}{\varkappa}\hbar c\sqrt{\frac{1}{\varepsilon'}}\,(\psi_N^\star \vec{\sigma}\,\psi_P \cdot \vec{k}')\right] \\ &\left[\frac{1}{2}e(\hbar c)^2\sqrt{\frac{1}{\hbar\nu}}\sqrt{\frac{1}{\varepsilon\varepsilon'}}\,(\vec{e}\vec{k} + \vec{e}\vec{k}')\right] \\ &-\frac{1}{\varepsilon'}\left[-\frac{1}{2}e(\hbar c)^2\sqrt{\frac{1}{\hbar\nu}}\sqrt{\frac{1}{\varepsilon\varepsilon''}}\,(\vec{e}\vec{k}'' - \vec{e}\vec{k})\right] \\ &\left[i\frac{f_2}{\varkappa}\hbar c\sqrt{\frac{1}{\varepsilon''}}\,(\psi_N^\star \vec{\sigma}\,\psi_P \cdot \vec{k}'')\right] \\ &-\frac{ie}{\hbar c}\frac{f_2}{\varkappa}\,(\psi_N^\star \vec{\sigma}\,\psi_P \cdot \vec{e})\,(\hbar c)^2\sqrt{\frac{1}{\hbar\nu}}\sqrt{\frac{1}{\varepsilon}} \end{aligned}\right\} \qquad (16)$$

where the unit vector $\vec{e}$ denotes the direction of polarization of the photon emitted, and ψ_N and ψ_P are the amplitudes of the plane waves representing the states of the neutron and the proton, respectively. From (15) we have $\vec{k}' = -\vec{k}''$ and $\varepsilon' = \varepsilon''$ and, denoting the energy $k\hbar c$ of the photon by ε, we get for the matrix element V the expression

$$V = i\frac{f_2}{\varkappa} e\hbar c \frac{\sqrt{2\pi}}{\varepsilon} \psi_N^\star \vec{\sigma} \psi_P \cdot \left[\frac{2\vec{k}'(\vec{k}\vec{e})}{k'^2 + \varkappa^2} - \vec{e} \right]. \qquad (17)$$

According to the usual perturbation theory, the probability per unit time of the emission of a photon into a solid angle $d\Omega_\nu$ is equal to

$$\frac{2\pi}{\hbar} \sum_{\vec{e}} |V|^2 \cdot \frac{\nu^2 d\Omega_\nu}{c^3} \left(\frac{d\nu}{d\varepsilon}\right) \qquad (18)$$

and, since

$$\frac{d\varepsilon}{d\nu} = h$$

we get, remembering that the nucleon momentum is small,

$$\frac{e^2}{\hbar^2 c}\left(\frac{f_2}{\varkappa}\right)^2 \frac{1}{\pi}\left[1 - \frac{2k^2\varkappa^2 \sin^2(\vec{k}\vec{\nu})}{(k'^2+\varkappa^2)^2}\right] d\Omega_\nu. \qquad (19)$$

An integration over all directions of the photon gives the final expression

$$4\frac{e^2}{\hbar^2 c}\left(\frac{f_2}{\varkappa}\right)^2 \left[1 + \frac{\varkappa^2}{k_\nu^2} + \frac{\varkappa^2}{k k_\nu} \log\frac{k_\nu - k}{\varkappa}\right] \qquad (20)$$

which, as may be seen, for large energies of the incident meson tends towards

$$4\frac{e^2}{\hbar^2 c}\left(\frac{f_2}{\varkappa}\right)^2. \qquad (20')$$

For the inverse process

$$N + h\nu \rightarrow P + M^- \qquad (21)$$

we get similar results. The probability per unit time for the absorption of the photon and the emission of the meson into a solid angle $d\Omega_k$ is, with the same notations as above, equal to

$$\frac{1}{2\pi}\,\frac{e^2}{\hbar^2 c}\left(\frac{f_2}{\varkappa}\right)^2 \sqrt{\frac{k^2}{k^2+\varkappa^2}}\left[1-\frac{4\varkappa^2(\vec{k}\,\vec{e})^2}{(k'^2+\varkappa^2)^2}\right] d\Omega_k \tag{22}$$

whence the total probability is obtained by integrating over $d\Omega_k$:

$$2\,\frac{e^2}{\hbar^2 c}\left(\frac{f_2}{\varkappa}\right)^2 \sqrt{\frac{k^2}{k^2+\varkappa^2}}\left[1+\frac{\varkappa^2}{k_\nu^2}+\frac{\varkappa^2}{k\,k_\nu}\log\frac{k_\nu-k}{\varkappa}\right]. \tag{23}$$

3. Scattering of mesons by nucleons.

For definiteness, we shall consider the process [6]

$$M_0^- + P \to P + M^- \tag{24 a}$$

the process

$$M_0^+ + N \to N + M^+ \tag{24 b}$$

being quite analogous.

Let $\vec{k}_0\hbar$ be the momentum of the incident meson M_0^-, $\vec{k}\hbar$ that of the scattered meson M^-, and ε_0 and ε the corresponding energies. Let, furthermore, ψ_0, ψ and ψ_N be the amplitudes of the plane waves representing the initial, final and intermediate (neutron) states of the nucleon.

Again we assume that the momenta of the nucleon in these states are small compared with Mc. The matrix element for the process turns out to be

$$V=\frac{1}{\varepsilon_0}\left[i\frac{f_2}{\varkappa}\hbar c\sqrt{\frac{1}{\varepsilon}}\,\psi^\star\vec{\sigma}\psi_N\cdot\vec{k}\right]\left[-i\frac{f_2}{\varkappa}\hbar c\sqrt{\frac{1}{\varepsilon_0}}\,\psi_N^\star\vec{\sigma}\psi_0\cdot\vec{k}_0\right] \tag{25}$$

whence, after summation over the possible spin states, we get

$$|V|^2_{\text{sum}} = \left(\frac{f_2}{\varkappa}\right)^4 k^2 k_0^2 \frac{1}{\sqrt{k^2+\varkappa^2}} \frac{1}{\sqrt{k_0^2+\varkappa^2}^3}. \tag{26}$$

Since the mass of the meson is small compared with M, the law of conservation of energy gives

$$k_0 = k \tag{27}$$

and, thus, we get for the total scattering probability per unit time the expression

$$\frac{1}{\pi} \frac{1}{\hbar^2 c} \left(\frac{f_2}{\varkappa}\right)^4 \frac{k_0^5}{\sqrt{k_0^2+\varkappa^2}^3}. \tag{28}$$

4. Multiple processes, by which a meson in a collision with a nucleon is transformed into a pair of mesons.

In this section, too, we confine ourselves to the consideration of the first of the two analogous processes

$$\left.\begin{aligned} M_0^+ + N &\rightarrow P + M^+ + M^- \\ M_0^- + P &\rightarrow N + M^+ + M^-. \end{aligned}\right\} \tag{29}$$

The process is a three-stage process which can take place in two different ways just as in the case of V-mesons [6]. Let the momenta and the energies of the mesons M_0^+, M^+ and M^- be $\vec{k}_0 \hbar$, $\vec{k}^+ \hbar$, $\vec{k}^- \hbar$, ε_0^+, ε^+ and ε^-, respectively. Under similar conditions as before, the law of conservation of energy gives

$$\varepsilon_0^+ = \varepsilon^+ + \varepsilon^-. \tag{30}$$

We denote the amplitudes of the wave-functions of the nucleons in the initial state by ψ_0, and in the final state by ψ. Similarly, the corresponding amplitudes of the intermediate states are denoted by ψ'_{I} and ψ'_{II} or ψ''_{I} and ψ''_{II}, according to the process taking place in the one or the other of the two ways mentioned above.

For the matrix element we get

$$\left.\begin{aligned} V &= \frac{1}{\varepsilon_0^+ (\varepsilon_0^+ - \varepsilon^+)} \left(-\frac{f_2}{\varkappa}\right)^3 \\ &\left\{\psi^\star \vec{\sigma} \psi'_{II} (-i\vec{k}^-) \cdot \frac{\hbar c}{\sqrt{\varepsilon^-}} \cdot \psi'^\star_{II} \vec{\sigma} \psi'_I (-i\vec{k}^+) \frac{\hbar c}{\sqrt{\varepsilon^+}} \psi'^\star_I \vec{\sigma} \psi_0 (i\vec{k}_0) \frac{\hbar c}{\sqrt{\varepsilon_0^+}}\right\} \\ &+ \frac{1}{(-\varepsilon^-)(-\varepsilon^- - \varepsilon^+)} \left(-\frac{f_2}{\varkappa}\right)^3 \\ &\left\{\psi^\star \vec{\sigma} \psi''_{II} \cdot i\vec{k}_0 \cdot \frac{\hbar c}{\sqrt{\varepsilon_0^+}} \cdot \psi''^\star_{II} \vec{\sigma} \psi''_I (-i\vec{k}^+) \frac{\hbar c}{\sqrt{\varepsilon^+}} \psi''^\star_I \vec{\sigma} \psi_0 (-i\vec{k}^-) \cdot \frac{\hbar c}{\sqrt{\varepsilon^-}}\right\}. \end{aligned}\right\} \quad (31)$$

The first term corresponds to the succession of events in which M_0^+ is first absorbed, and M^+ and M^- are then emitted, while the second term is connected with a process in which the absorption of M_0^+ takes place after the emission of M^+ and M^-.

After the summation over the possible spin states is carried out, we obtain

$$\left.\begin{aligned} |V|^2_{\text{sum}} &= \left(\frac{f_2}{\varkappa}\right)^6 (\hbar c)^6 \, 2 \frac{1}{(\varepsilon^- \varepsilon_0^+)^2} \frac{1}{\varepsilon^- \varepsilon^+ \varepsilon_0^+} \cdot \\ \cdot &\left\{ (\vec{k}_0^+ \vec{k}^+)^2 (k^-)^2 + (\vec{k}^+ \vec{k}^-)^2 k_0^2 + (\vec{k}^- \vec{k}_0^+)^2 (k^+)^2 - \right. \\ &\left. - 2 (\vec{k}_0^+ \vec{k}^+) (\vec{k}^+ \vec{k}^-) (\vec{k}^- \vec{k}_0^+) \right\}. \end{aligned}\right\} \quad (32)$$

The probability per unit time of the process taking place is, as usual, given by the expression

$$\frac{2\pi}{\hbar} |V|^2_{\text{sum}} \frac{k^{+2} k^{-2} \, d\Omega^+ \, d\Omega^-}{(8\pi^3)^2} \, dk^- \frac{dk^+}{d\varepsilon}. \quad (33)$$

Substituting the value of (32) and performing the integration over the angles $d\Omega^+$ and $d\Omega^-$ in the direction of the momenta $\vec{k}^+$ and $\vec{k}^-$, we are finally led to the expression

$$\frac{7}{9}\frac{2}{\pi^3}\frac{1}{\hbar}\left(\frac{f_2}{\varkappa}\right)^6\frac{1}{\hbar^2 c^2}\frac{(k_0^+)^2 (k^+)^3 (k^-)^4 \, dk^-}{\left\{(k_0^+)^2+\varkappa^2\right\}^{3/2}\left\{(k^-)^2+\varkappa^2\right\}^{3/2}} \tag{34}$$

for the probability per unit time for the emission of the meson M^- with a momentum lying between k^- and $k^- + dk^-$, the momentum k^+ in this last formula being connected with k^- by the formula (30).

5. The lifetime of a free *PS*-meson.

According to the fundamental assumptions of the meson theory, a meson may disintegrate spontaneously into an electron and a neutrino. This assumption directly verified by experiments [7] is also of importance for the theory of β-disintegration both as regards the V-mesons [8] and the PS-mesons [9]. In order to calculate the probability of disintegration of a PS-meson [10] we have to consider the terms in the Hamiltonian which describe the interaction between PS-mesons and light particles. These terms are obtained by replacing in the Hamiltonian (5) the quantities $\vec{P}$, Q, R by the analogous quantities(1)

$$\left.\begin{aligned}
\breve{\vec{P}} &= \frac{\breve{f}_2}{\varkappa}\,\breve{\psi}^\star\,\breve{\vec{\sigma}}\,\frac{\breve{\tau}_1 - i\breve{\tau}_2}{2}\,\breve{\psi}\\
\breve{Q} &= \frac{\breve{f}_2}{\varkappa}\,\breve{\psi}^\star\,\breve{\varrho}_1\,\frac{\breve{\tau}_1 - i\breve{\tau}_2}{2}\,\breve{\psi}\\
\breve{R} &= \breve{f}_1\,\breve{\psi}^\star\,\breve{\varrho}_2\,\frac{\breve{\tau}_1 - i\breve{\tau}_2}{2}\,\breve{\psi}_1
\end{aligned}\right\} \tag{35}$$

where, in general, the symbol $\breve{A}$ denotes the same quantity with respect to light particles as A with respect to nucleons.

(1) It should be remarked that the constants $\breve{f}_1$ and $\breve{f}_2$ in this paper are chosen in a different way as those in ref. [9], so that the ratio between these constants is of opposite sign.

In the process in question a negative meson, say, with the momentum $\vec{k}\hbar$ will be annihilated, raising at the same time a light particle from a neutrino-state with the momentum $\vec{p}_\sigma$, the energy E_σ, and described by the wave-function $\psi_\sigma e^{\frac{i}{\hbar}(\vec{p}_\sigma \vec{x})}$ to an electron state with the momentum $\vec{p}_s$, the energy E_s and described by the wave-function $\psi_s e^{\frac{i}{\hbar}(\vec{p}_s \vec{x})}$. The corresponding matrix element is evidently given by

$$V = -\frac{\breve{f}_2}{\varkappa} i \frac{\hbar c}{\sqrt{\varepsilon}} \psi_s^\star \vec{\breve{\sigma}} \psi_\sigma \cdot \vec{k} - \frac{\breve{f}_2}{\varkappa} i \sqrt{\varepsilon}\, \psi_s^\star \breve{\varrho}_1 \psi_\sigma - \breve{f}_1 \frac{\hbar c}{\sqrt{\varepsilon}} \psi_s^\star \breve{\varrho}_2 \psi_\sigma . \quad (36)$$

From the laws of conservation of momentum and energy it follows that

$$\vec{p}_\sigma + \vec{k}\hbar = \vec{p}_s \quad (37)$$

and

$$\hbar c \sqrt{k^2 + \varkappa^2} + E_\sigma = E_s . \quad (38)$$

For the frame of reference in which the meson is at rest ($\vec{k} = 0$, $\vec{p}_s = \vec{p}_\sigma$), the above expression for V reduces to

$$V = -\sqrt{\frac{\hbar c}{\varkappa}} \left(\breve{f}_2\, i\, \psi_s^\star \breve{\varrho}_1 \psi_s + \breve{f}_1 \psi_s^\star \breve{\varrho}_2 \psi_\sigma \right) . \quad (39)$$

Squaring and summing over the possible states of the electron and the neutrino, we get

$$\left. \begin{aligned} |V|^2_{\text{sum}} &= \sqrt{\frac{\hbar c}{\varkappa}} \frac{1}{E_s E_\sigma} \\ \{E_s E_\sigma (\breve{f}_1^2 + \breve{f}_2^2) &+ 2 E_\sigma mc^2 \breve{f}_1 \breve{f}_2 + c^2 (\vec{p}_s \vec{p}_\sigma) (\breve{f}_2^2 - \breve{f}_1^2)\} . \end{aligned} \right\} \quad (40)$$

Substituting this value into the expression for the probability per unit time of disintegration of the meson

$$\frac{2\pi}{\hbar} \int \frac{p_\sigma^2\, dp_\sigma\, d\Omega}{h^3} \, |V|^2_{\text{sum}} \, \delta(E_s - E_\sigma - \hbar c \varkappa) \quad (41)$$

we obtain for the decay constant λ_{PS} of a *PS*-meson the formula

$$\lambda_{PS} = \frac{\varkappa}{4\pi\hbar}\left(1 - \frac{m^2 c^2}{\varkappa^2 \hbar^2}\right)^2 \left(\breve{f}_1 + \frac{mc}{\varkappa\hbar}\breve{f}_2\right)^2. \tag{42}$$

The lifetime $\tau_{PS} = 1/\lambda_{PS}$ thus found is that of a meson at rest. For a meson moving with velocity v the lifetime will be given by the expression

$$\frac{\tau_{PS}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

6. Numerical estimates.

For a numerical estimate of the probabilities or the corresponding cross-sections (the probability divided by the velocity of the incident particles) it is necessary to know the value of the constant f_2. Taking the value determined from the theory of nuclear forces (ref. [1], formulae (54) and (107)), and putting

$$\frac{1}{\varkappa} = \frac{1}{150}\frac{\hbar}{mc} \backsim 2{,}6 \times 10^{-13}\ \mathrm{cm}$$

we get for the process 2 a cross-section of the order of $4{,}5 \cdot 10^{-27}$ cm^2 for $k = 0{,}5\,\varkappa$ and $2{,}0 \cdot 10^{-27}$ cm^2 for $k = 2\,\varkappa$. For the process 3, the cross-section is $1{,}0 \cdot 10^{-27}$ cm^2 for $k_0 = 0{,}5\,\varkappa$, $6{,}5 \cdot 10^{-26}$ cm^2 for $k_0 = 2\,\varkappa$ and $5 \cdot 10^{-25}$ cm^2 for $k_0 = 5\,\varkappa$. Finally, the cross-section for the process 4 is of the order of 10^{-31} cm^2 for $k_0 = 2\,\varkappa$, and 10^{-26} cm^2 for $k_0 = 5\,\varkappa$.

The lifetime of the *PS*-mesons calculated in Section 5 depends upon the constants $\breve{f}_1$ and $\breve{f}_2$. For the discussion of this question and its connection with the lifetimes of β-radioactive elements, compare ref. [5].

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